Topic 2 – Functions and equations

Aims

The aims of this section are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of a GDC in both the development and the application of this topic.

2.1 Concept of function $f : x \mapsto f(x)$: domain, range; image (value).

On examination papers: if the domain is the set of real numbers then the statement "$x \in \mathbb{R}$" will be omitted.

Exclusion: Formal definition of a function; the terms "one-to-one", "many-to-one" and "codomain".

Composite functions $f \circ g$; identity function.

The composite function $(f \circ g)(x)$ is defined as $f(g(x))$.

Inverse function f^{-1}.

On examination papers: if an inverse function is to be found, the given function will be defined with a domain that ensures it is one-to-one.

Exclusion: Domain restriction.

2.2 The graph of a function; its equation $y = f(x)$.

On examination papers: questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus.

The linear function $ax + b$ is now in the presumed knowledge section.

Function graphing skills:

use of a GDC to graph a variety of functions;

investigation of key features of graphs.

Identification of horizontal and vertical asymptotes.

Solution of equations graphically.

May be referred to as roots of equations, or zeros of functions.

2.3 Transformations of graphs: translations; stretches; reflections in the axes.

Translations: $y = f(x) + b$; $y = f(x - a)$.

Stretches: $y = pf(x)$; $y = f(x/q)$.

Reflections (in both axes): $y = -f(x)$; $y = f(-x)$.

Examples: $y = x^2$ *used to obtain* $y = 3x^2 + 2$ *by a stretch of scale factor 3 in the y-direction followed by a translation of* $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

$y = \sin x$ *used to obtain* $y = 3\sin 2x$ *by a stretch of scale factor 3 in the y-direction and a stretch of scale factor* $\frac{1}{2}$ *in the x-direction.*

The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.

2.4 The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph; its self-inverse nature.

2.5 The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$.

Rational coefficients only.

Axis of symmetry $x = -\frac{b}{2a}$.

The form $x \mapsto a(x-h)^2 + k$: vertex (h, k).

The form $x \mapsto a(x-p)(x-q)$: x-intercepts $(p, 0)$ and $(q, 0)$.

2.6 The solution of $ax^2 + bx + c = 0$, $a \neq 0$.

Exclusion: On examination papers: questions demanding elaborate factorization techniques will not be set.

The quadratic formula.

Use of the discriminant $\Delta = b^2 - 4ac$.

2.7 The function: $x \mapsto a^x$, $a > 0$.

The inverse function $x \mapsto \log_a x$, $x > 0$.

$\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.

Graphs of $y = a^x$ and $y = \log_a x$.

Solution of $a^x = b$ using logarithms.

2.8 The exponential function $x \mapsto e^x$.

The logarithmic function $x \mapsto \ln x$, $x > 0$.

$a^x = e^{x \ln a}$.

Examples of applications: compound interest, growth and decay.