

REPASO 2

Nombre:

FECHA:

1. The mass  $m$  kg of a radio-active substance at time  $t$  hours is given by

$$m = 4e^{-0.2t}.$$

- (a) Write down the initial mass.  
(b) The mass is reduced to 1.5 kg. How long does this take?

**(Total 6 marks)**

2. The function  $f$  is given by  $f(x) = x^2 - 6x + 13$ , for  $x \geq 3$ .

- (a) Write  $f(x)$  in the form  $(x - a)^2 + b$ .  
(b) Find the inverse function  $f^{-1}$ .  
(c) State the domain of  $f^{-1}$ .

**(Total 6 marks)**

3. The equation  $kx^2 + 3x + 1 = 0$  has exactly one solution. Find the value of  $k$ .

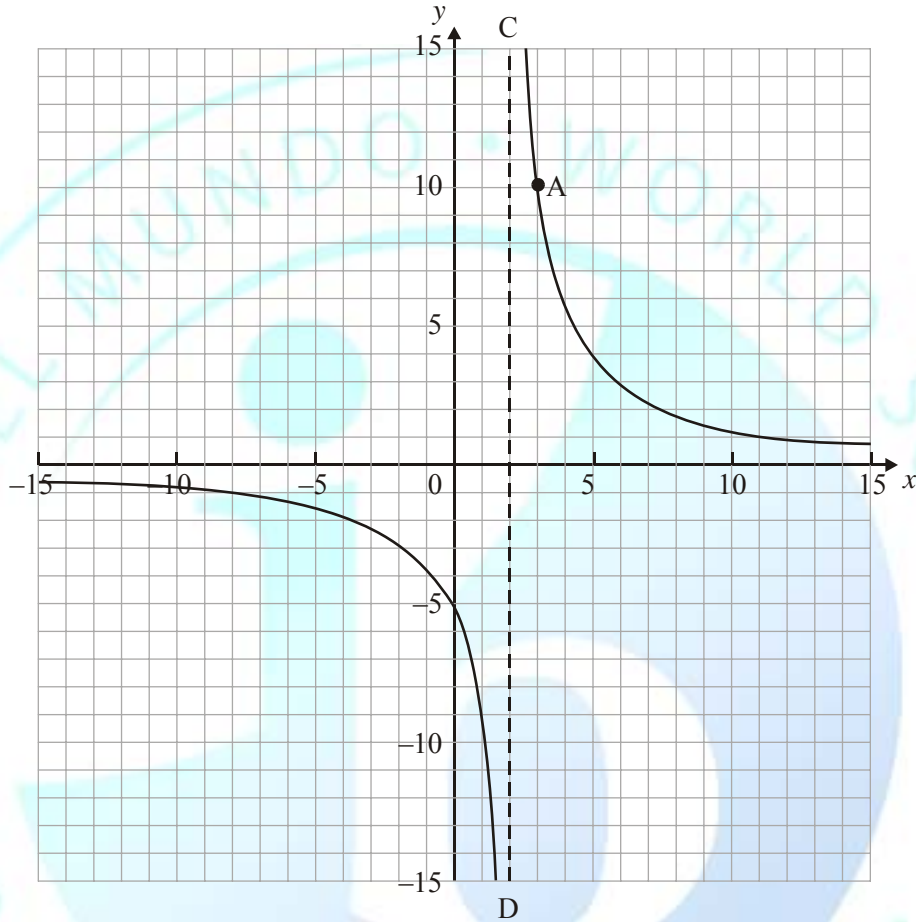
**(Total 6 marks)**

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4. (a) The diagram shows part of the graph of the function  $f(x) = \frac{q}{x-p}$ . The curve passes through the point A (3, 10). The line (CD) is an asymptote.



Find the value of

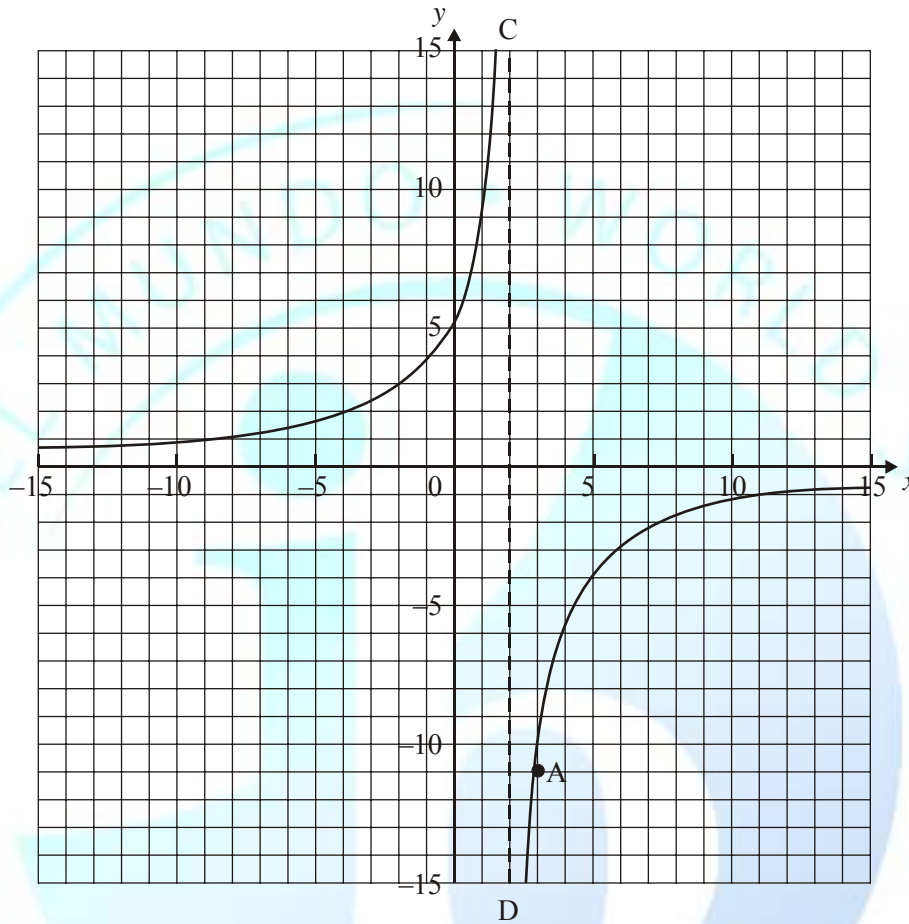
- (i)  $p$ ;
- (ii)  $q$ .

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- (b) The graph of  $f(x)$  is transformed as shown in the following diagram. The point A is transformed to  $A'(3, -10)$ .



Give a full geometric description of the transformation.

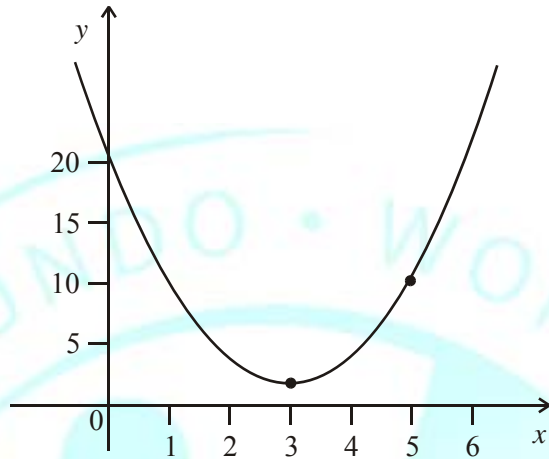
(Total 6 marks)

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5. The diagram shows part of the graph of the curve  $y = a(x - h)^2 + k$ , where  $a, h, k \in \mathbb{Z}$ .



- (a) The vertex is at the point (3, 1). Write down the value of  $h$  and of  $k$ . (2)
- (b) The point P(5, 9) is on the graph. Show that  $a = 2$ . (3)
- (c) Hence show that the equation of the curve can be written as  

$$y = 2x^2 - 12x + 19.$$
 (1)
- (d) (i) Find  $\frac{dy}{dx}$ .
- A tangent is drawn to the curve at P (5, 9).
- (ii) Calculate the gradient of this tangent,
- (iii) Find the equation of this tangent. (4)

(Total 10 marks)

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6. A family of functions is given by

$$f(x) = x^2 + 3x + k, \text{ where } k \in \{1, 2, 3, 4, 5, 6, 7\}.$$

One of these functions is chosen at random. Calculate the probability that the curve of this function crosses the  $x$ -axis.

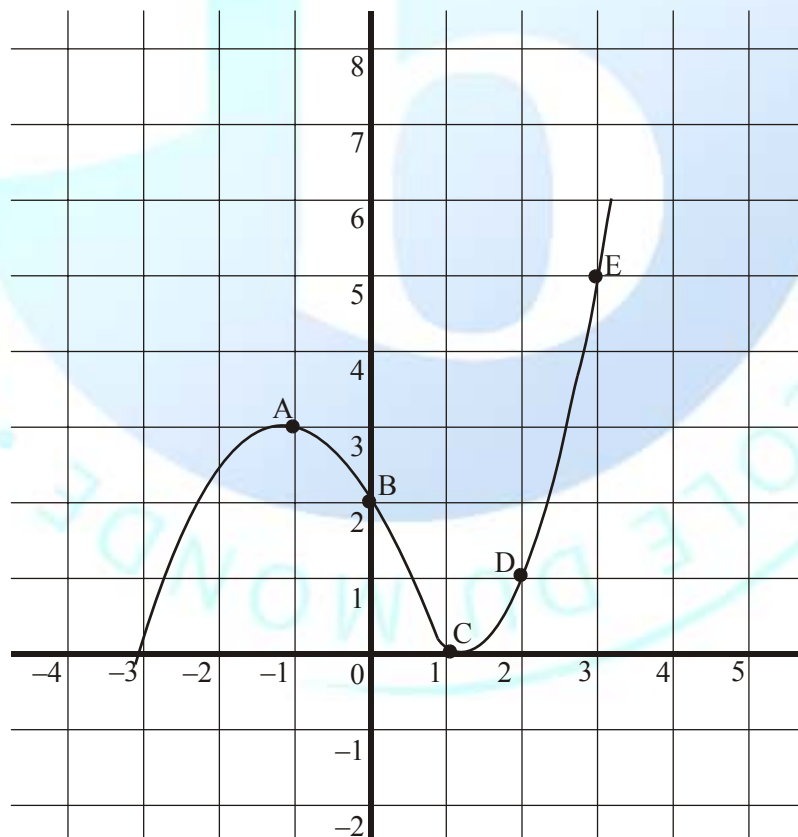
(Total 6 marks)

7. Let  $f(x) = e^{-x}$  and  $g(x) = \frac{x}{1+x}, x \neq -1$ . Find

- (a)  $f^{-1}(x)$   
(b)  $(g \circ f)(x)$ .

(Total 6 marks)

8. The sketch shows part of the graph of  $y = f(x)$  which passes through the points A(-1, 3), B(0, 2), C(1, 0), D(2, 1) and E(3, 5).



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A second function is defined by  $g(x) = 2f(x - 1)$ .

- (a) Calculate  $g(0)$ ,  $g(1)$ ,  $g(2)$  and  $g(3)$ .
- (b) On the same axes, sketch the graph of the function  $g(x)$ .

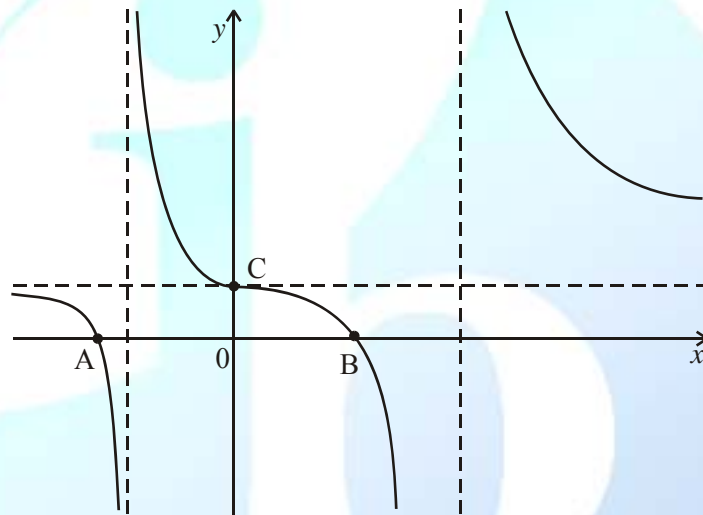
(Total 6 marks)

9. Let  $g(x) = x^4 - 2x^3 + x^2 - 2$ .

- (a) Solve  $g(x) = 0$ .

(2)

Let  $f(x) = \frac{2x^3}{g(x)} + 1$ . A part of the graph of  $f(x)$  is shown below.



- (b) The graph has vertical asymptotes with equations  $x = a$  and  $x = b$  where  $a < b$ . Write down the values of

- (i)  $a$ ;
- (ii)  $b$ .

(2)

- (c) The graph has a horizontal asymptote with equation  $y = 1$ . Explain why the value of  $f(x)$  approaches 1 as  $x$  becomes very large.

(2)

- (d) The graph intersects the  $x$ -axis at the points A and B. Write down the **exact** value of the  $x$ -coordinate at

- (i) A;
- (ii) B.

(2)

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- (e) The curve intersects the  $y$ -axis at  $C$ . **Use the graph** to explain why the values of  $f(x)$  and  $f''(x)$  are zero at  $C$ .

(2)

(Total 10 marks)

10. Given that  $f(x) = 2e^{3x}$ , find the inverse function  $f^{-1}(x)$ .

(Total 4 marks)

11. Consider the functions  $f: x \mapsto 4(x-1)$  and  $g: x \mapsto \frac{6-x}{2}$ .

- (a) Find  $g^{-1}$ .  
(b) Solve the equation  $(f \circ g^{-1})(x) = 4$ .

(Total 6 marks)

12. Solve the equation  $e^x = 5 - 2x$ , giving your answer correct to **four** significant figures.

(Total 6 marks)

13. Consider the function  $f(x) = 2x^2 - 8x + 5$ .

- (a) Express  $f(x)$  in the form  $a(x-p)^2 + q$ , where  $a, p, q \in \mathbb{Z}$ .  
(b) Find the minimum value of  $f(x)$ .

(Total 6 marks)

14. Consider functions of the form  $y = e^{-kx}$

- (a) Show that  $\int_0^1 e^{-kx} dx = \frac{1}{k} (1 - e^{-k})$ .

(3)

- (b) Let  $k = 0.5$

- (i) Sketch the graph of  $y = e^{-0.5x}$ , for  $-1 \leq x \leq 3$ , indicating the coordinates of the  $y$ -intercept.  
(ii) Shade the region enclosed by this graph, the  $x$ -axis,  $y$ -axis and the line  $x = 1$ .

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(iii) Find the area of this region.

(5)

(c) (i) Find  $\frac{dy}{dx}$  in terms of  $k$ , where  $y = e^{-kx}$ .

The point  $P(1, 0.8)$  lies on the graph of the function  $y = e^{-kx}$ .

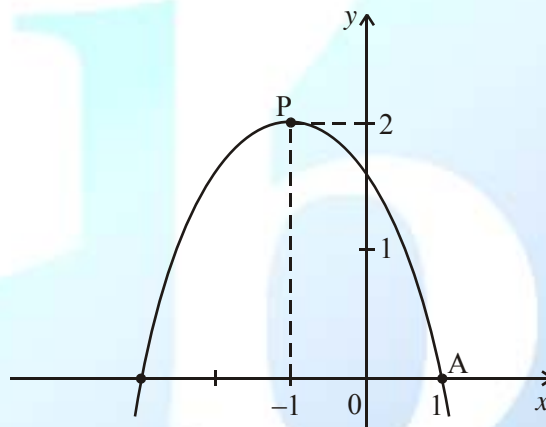
(ii) Find the value of  $k$  in this case.

(iii) Find the gradient of the tangent to the curve at  $P$ .

(5)

(Total 13 marks)

15. The diagram shows part of the graph of  $y = a(x - h)^2 + k$ . The graph has its vertex at  $P$ , and passes through the point  $A$  with coordinates  $(1, 0)$ .



(a) Write down the value of

(i)  $h$ ;

(ii)  $k$ .

(b) Calculate the value of  $a$ .

(Total 6 marks)

16. Let  $f(x) = 2^x$ , and  $g(x) = \frac{x}{x-2}$ , ( $x \neq 2$ ).



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Find

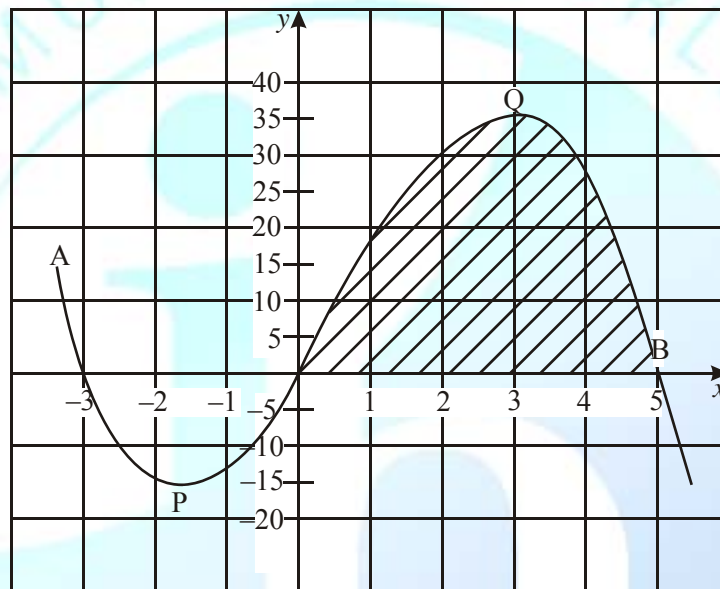
(a)  $(g \circ f)(3)$ ;

(b)  $g^{-1}(5)$ .

(Total 6 marks)

17. The diagram below shows part of the graph of the function

$$f : x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the  $x$ -axis at  $A(-3,0)$ ,  $B(5,0)$  and the origin,  $O$ . There is a minimum point at  $P$  and a maximum point at  $Q$ .

(a) The function may also be written in the form  $f : x \mapsto -x(x-a)(x-b)$ , where  $a < b$ . Write down the value of

(i)  $a$ ;

(ii)  $b$ .

(2)

(b) Find

(i)  $f'(x)$ ;

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(ii) the **exact** values of  $x$  at which  $f'(x) = 0$ ;

(iii) the value of the function at Q.

(7)

(c) (i) Find the equation of the tangent to the graph of  $f$  at O.

(ii) This tangent cuts the graph of  $f$  at another point. Give the  $x$ -coordinate of this point.

(4)

(d) Determine the area of the shaded region.

(2)

(Total 15 marks)

18. In the diagram below, the points  $O(0, 0)$  and  $A(8, 6)$  are fixed. The angle  $\widehat{OPA}$  varies as the point  $P(x, 10)$  moves along the horizontal line  $y = 10$ .

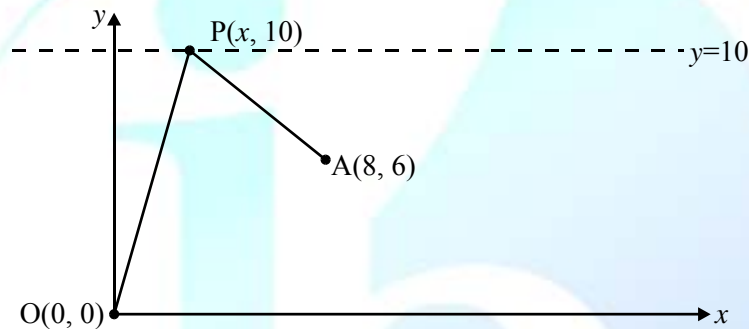


diagram to scale

(a) (i) Show that  $AP = \sqrt{x^2 - 16x + 80}$ .

(ii) Write down a similar expression for OP in terms of  $x$ .

(2)

(b) Hence, show that

$$\cos \widehat{OPA} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$$

(3)

(c) Find, in degrees, the angle  $\widehat{OPA}$  when  $x = 8$ .

(2)

(d) Find the positive value of  $x$  such that  $\widehat{OPA} = 60^\circ$ .

(4)

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Let the function  $f$  be defined by

$$f(x) = \cos \widehat{OPA} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}, \quad 0 \leq x \leq 15.$$

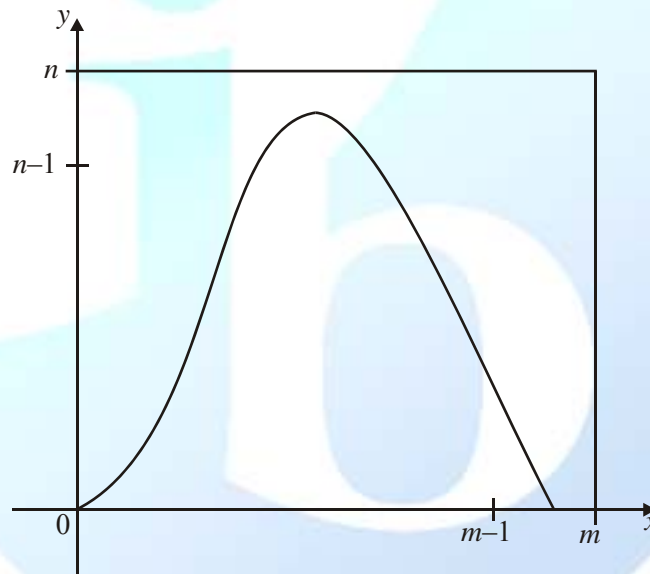
(e) Consider the equation  $f(x) = 1$ .

- (i) Explain, in terms of the position of the points O, A, and P, why this equation has a solution.
- (ii) Find the **exact** solution to the equation.

(5)

(Total 16 marks)

19. The diagram below shows the graph of  $y = x \sin \frac{x}{3}$ , for  $0 \leq x < m$ , and  $0 \leq y < n$ , where  $x$  is in radians and  $m$  and  $n$  are integers.



Find the value of

- (a)  $m$ ;
- (b)  $n$ .

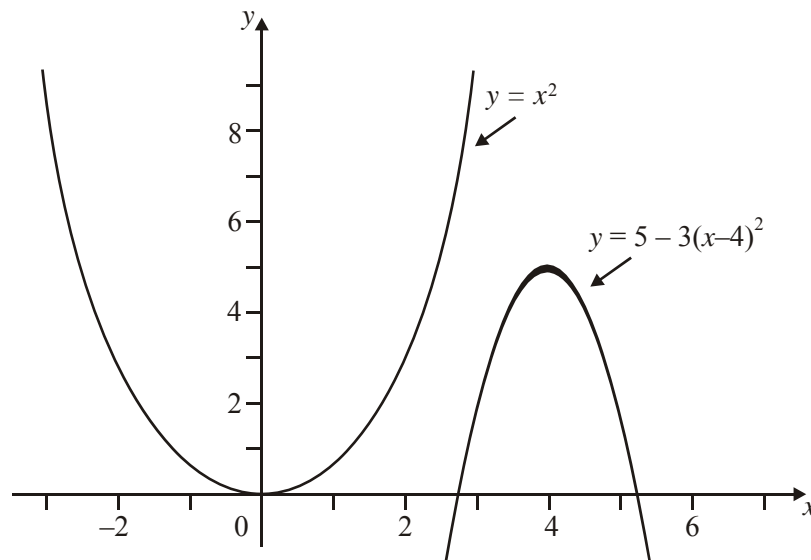
(Total 4 marks)

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20. The diagram shows parts of the graphs of  $y = x^2$  and  $y = 5 - 3(x - 4)^2$ .



The graph of  $y = x^2$  may be transformed into the graph of  $y = 5 - 3(x - 4)^2$  by these transformations.

A reflection in the line  $y = 0$  followed by  
 a vertical stretch with scale factor  $k$  followed by  
 a horizontal translation of  $p$  units followed by  
 a vertical translation of  $q$  units.

Write down the value of

- (a)  $k$ ;
- (b)  $p$ ;
- (c)  $q$ .

(Total 4 marks)

21. (a) Factorise  $x^2 - 3x - 10$ .
- (b) Solve the equation  $x^1 - 3x - 10 = 0$ .

(Total 4 marks)

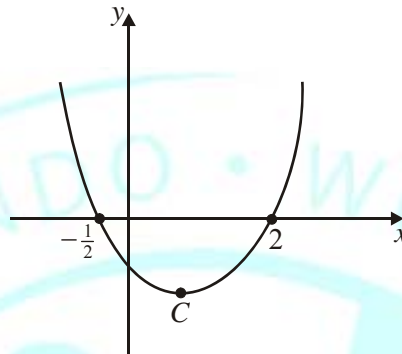
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22. The diagram represents the graph of the function

$$f: x \mapsto (x - p)(x - q).$$

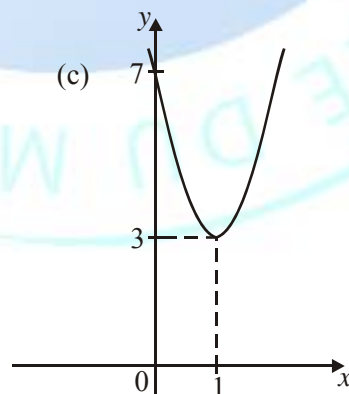
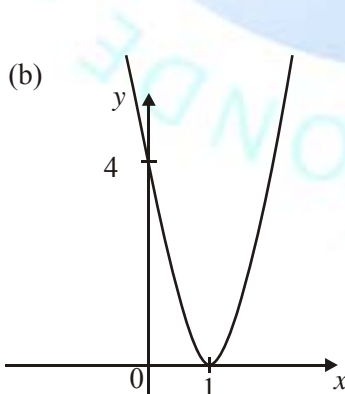
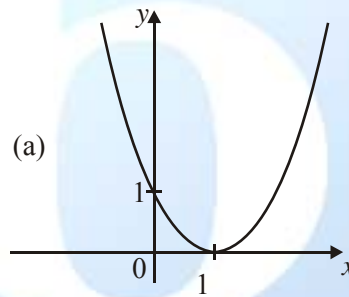
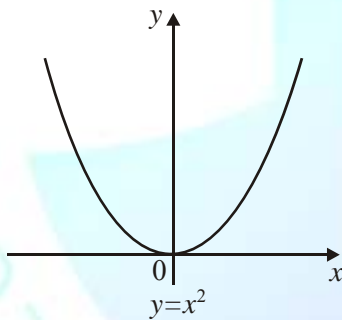


- (a) Write down the values of  $p$  and  $q$ .  
 (b) The function has a minimum value at the point  $C$ . Find the  $x$ -coordinate of  $C$ .

**Total 4 marks)**

23. The diagrams show how the graph of  $y = x^2$  is transformed to the graph of  $y = f(x)$  in three steps.

For each diagram give the equation of the curve.



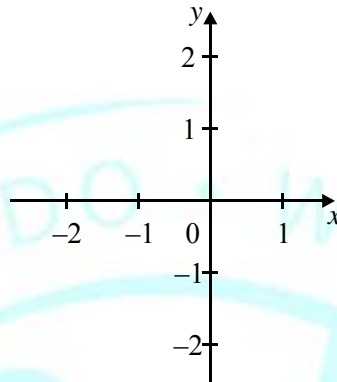
(Total 4 marks)

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24. (a) On the following diagram, sketch the graphs of  $y = e^x$  and  $y = \cos x$  for  $-2 \leq x \leq 1$ .



- (b) The equation  $e^x = \cos x$  has a solution between  $-2$  and  $-1$ .  
Find this solution.

(Total 4 marks)

25. The function  $f$  is defined by

$$f: x \mapsto \sqrt{3-2x}, \quad x \leq \frac{3}{2}.$$

Evaluate  $f^{-1}(5)$ .

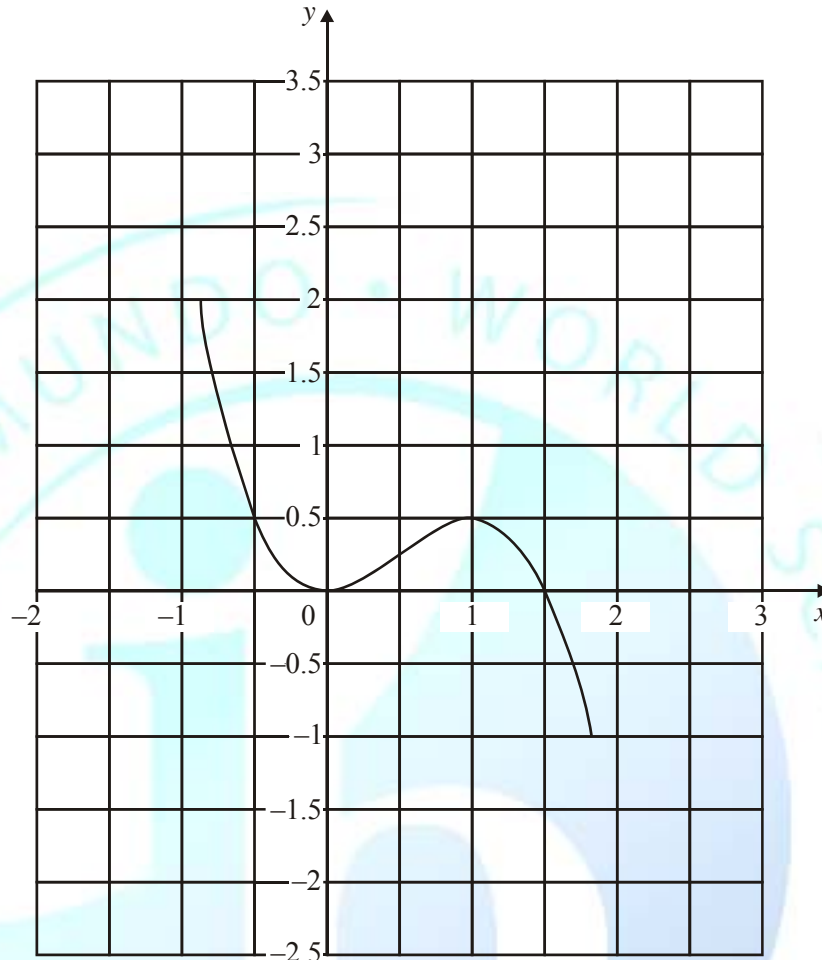
(Total 4 marks)

26. The following diagram shows the graph of  $y = f(x)$ . It has minimum and maximum points at

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$(0, 0)$  and  $(1, \frac{1}{2})$ .

(a) On the same diagram, draw the graph of  $y = f(x-1) + \frac{3}{2}$ .

(b) What are the coordinates of the minimum and maximum points of  $y = f(x-1) + \frac{3}{2}$ ?

**(Total 4 marks)**

27. Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc.

**(3)**

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- (b) How many complete years will it take for Michele's initial investment to double in value? (3)
- (c) What should the interest rate be if Michele's initial investment were to double in value in 10 years? (4)
- (Total 10 marks)

28. *Note: Radians are used throughout this question.*

Let  $f(x) = \sin(1 + \sin x)$ .

- (a) (i) Sketch the graph of  $y = f(x)$ , for  $0 \leq x \leq 6$ .  
(ii) Write down the  $x$ -coordinates of all minimum and maximum points of  $f$ , for  $0 \leq x \leq 6$ . Give your answers correct to **four** significant figures. (9)
- (b) Let  $S$  be the region in the first quadrant completely enclosed by the graph of  $f$  and **both** coordinate axes.  
(i) Shade  $S$  on your diagram.  
(ii) Write down the integral which represents the area of  $S$ .  
(iii) Evaluate the area of  $S$  to **four** significant figures. (5)
- (c) Give reasons why  $f(x) \geq 0$  for all values of  $x$ . (2)
- (Total 16 marks)



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29.  $f(x) = 4 \sin\left(3x + \frac{\pi}{2}\right)$ .

For what values of  $k$  will the equation  $f(x) = k$  have no solutions?

(Total 4 marks)

30. A group of ten leopards is introduced into a game park. After  $t$  years the number of leopards,  $N$ , is modelled by  $N = 10 e^{0.4t}$ .

- (a) How many leopards are there after 2 years?
- (b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

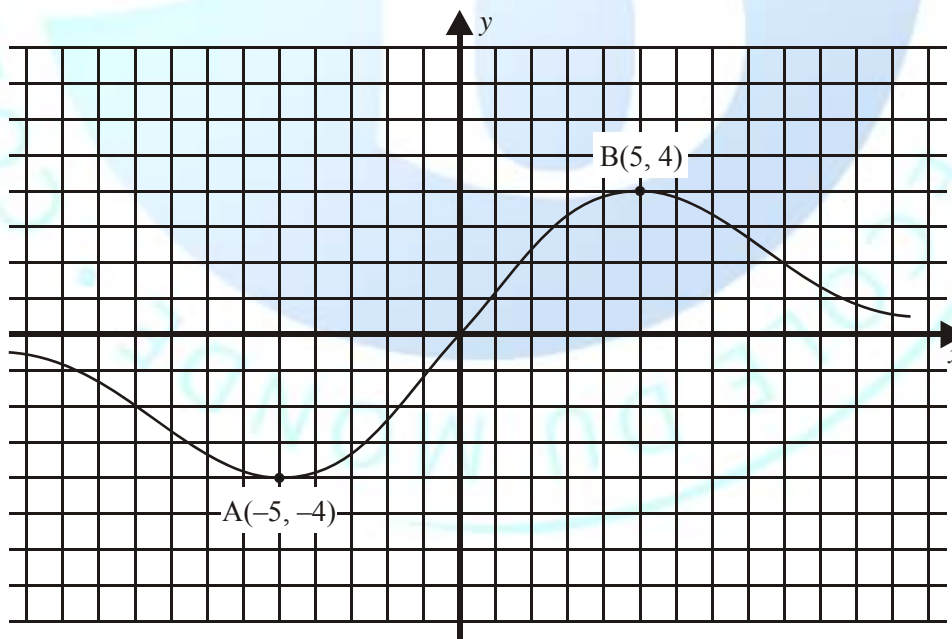
(Total 4 marks)

31. Consider the function  $f: x \mapsto \sqrt{x+1}$ ,  $x \geq -1$

- (a) Determine the inverse function  $f^{-1}$ .
- (b) What is the domain of  $f^{-1}$ ?

(Total 4 marks)

32. The diagram shows the graph of  $y = f(x)$ , with the  $x$ -axis as an asymptote.



- (a) On the same axes, draw the graph of  $y = f(x+2) - 3$ , indicating the coordinates of the

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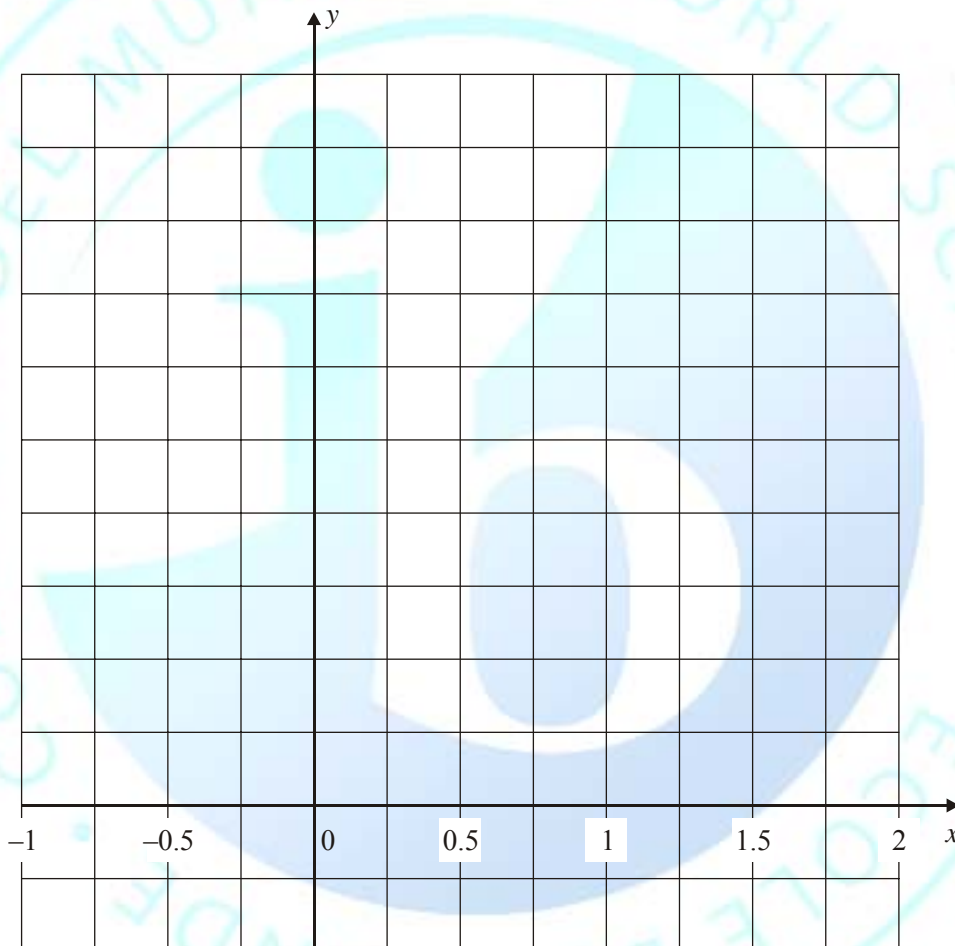
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- images of the points A and B.  
(b) Write down the equation of the asymptote to the graph of  $y = f(x + 2) - 3$ .

(Total 4 marks)

33. (a) Sketch, on the given axes, the graphs of  $y = x^2$  and  $y = \sin x$  for  $-1 \leq x \leq 2$ .



- (b) Find the positive solution of the equation

$$x^2 = \sin x,$$

giving your answer correct to 6 significant figures.

(Total 4 marks)

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34. A ball is thrown vertically upwards into the air. The height,  $h$  metres, of the ball above the ground after  $t$  seconds is given by

$$h = 2 + 20t - 5t^2, t \geq 0$$

- (a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released). (2)
- (b) Show that the height of the ball after one second is 17 metres. (2)
- (c) At a later time the ball is **again** at a height of 17 metres.
- (i) Write down an equation that  $t$  must satisfy when the ball is at a height of 17 metres.
- (ii) Solve the equation **algebraically**. (4)
- (d) (i) Find  $\frac{dh}{dt}$ .
- (ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).
- (iii) Find **when** the ball reaches its maximum height.
- (iv) Find the maximum height of the ball. (7)

(Total 15 marks)

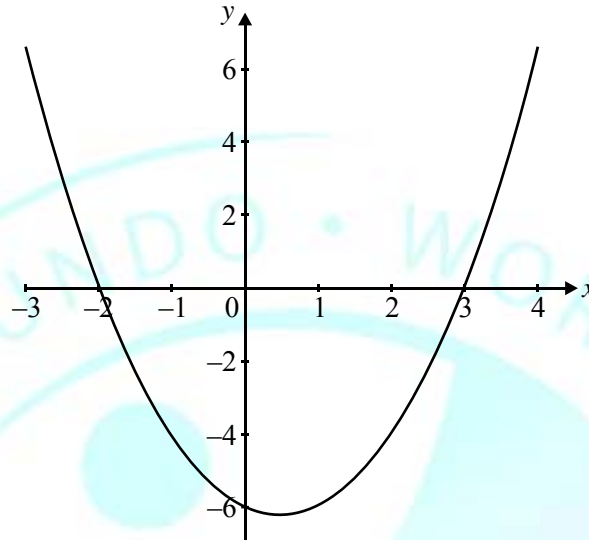
35. The diagram shows part of the graph with equation  $y = x^2 + px + q$ . The graph cuts the  $x$ -axis at

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-2 and 3.



Find the value of

- (a)  $p$ ;
- (b)  $q$ .

(Total 4 marks)

36. Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.

- (a) What was the population one year ago?
- (b) What was the population five years ago?

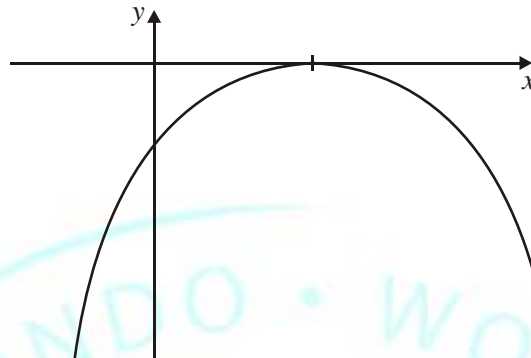
(Total 4 marks)

37. The diagram shows the graph of the function  $y = ax^2 + bx + c$ .

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Complete the table below to show whether each expression is positive, negative or zero.

Expression	positive	negative	zero
$a$			
$c$			
$b^2 - 4ac$			
$b$			

(Total 4 marks)

38. Initially a tank contains 10 000 litres of liquid. At the time  $t = 0$  minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid,  $V$  litres, which remains in the tank after  $t$  minutes is given by

$$V = 10\,000(0.933^t).$$

- (a) Find the value of  $V$  after 5 minutes. (1)
- (b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank. (3)
- (c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen. (3)
- (d) (i) Find the value of  $10\,000 - V$  when  $t = 0.001$  minutes. (1)
- (ii) Hence or otherwise, estimate the initial flow rate of the liquid. Give your answer in litres per minute, correct to two significant figures. (3)

(Total 10 marks)

39. (a) Express  $f(x) = x^2 - 6x + 14$  in the form  $f(x) = (x - h)^2 + k$ , where  $h$  and  $k$  are to be determined.

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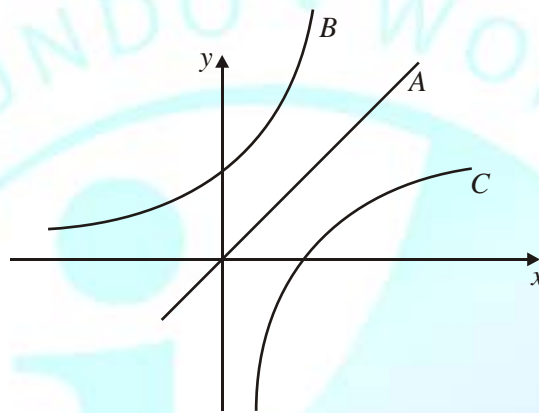
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- (b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation  $y = x^2 - 6x + 14$ .

(Total 4 marks)

40. The diagram shows three graphs.



A is part of the graph of  $y = x$ .

B is part of the graph of  $y = 2^x$ .

C is the reflection of graph B in line A.

Write down

- (a) the equation of C in the form  $y = f(x)$ ;  
(b) the coordinates of the point where C cuts the x-axis.

(Total 4 marks)

41. The quadratic equation  $4x^2 + 4kx + 9 = 0$ ,  $k > 0$  has exactly one solution for  $x$ . Find the value of  $k$ .

(Total 4 marks)

42. Two functions  $f, g$  are defined as follows:

$$f: x \rightarrow 3x + 5$$

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$$g : x \rightarrow 2(1 - x)$$

Find

- (a)  $f^{-1}(2)$ ;  
(b)  $(g \circ f)(-4)$ .

(Total 4 marks)

43. Two functions  $f$  and  $g$  are defined as follows:

$$f(x) = \cos x, \quad 0 \leq x \leq 2\pi;$$

$$g(x) = 2x + 1, \quad x \in \mathbb{R}.$$

Solve the equation  $(g \circ f)(x) = 0$ .

(Total 4 marks)

44. The function  $f$  is given by

$$F(x) = \frac{2x+1}{x-3}, \quad x \in \mathbb{R}, x \neq 3.$$

- (a) (i) Show that  $y = 2$  is an asymptote of the graph of  $y = f(x)$ . (2)  
(ii) Find the vertical asymptote of the graph. (1)  
(iii) Write down the coordinates of the point  $P$  at which the asymptotes intersect. (1)
- (b) Find the points of intersection of the graph and the axes. (4)
- (c) Hence sketch the graph of  $y = f(x)$ , showing the asymptotes by dotted lines. (4)
- (d) Show that  $f'(x) = \frac{-7}{(x-3)^2}$  and hence find the equation of the tangent at the point  $S$  where  $x = 4$ . (6)
- (e) The tangent at the point  $T$  on the graph is parallel to the tangent at  $S$ .  
Find the coordinates of  $T$ . (5)

REPASO 2

Nombre:

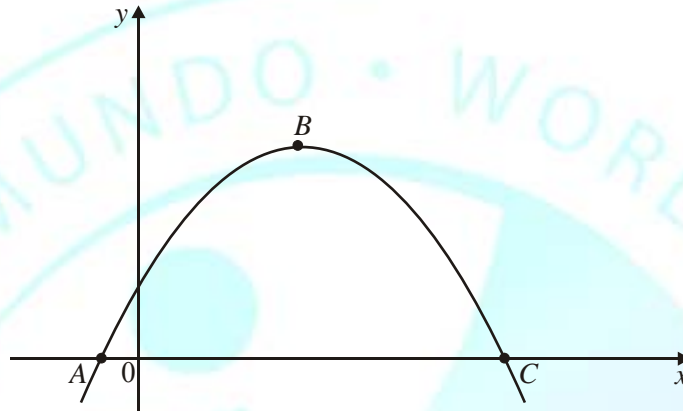
FECHA:

- (f) Show that  $P$  is the midpoint of  $[ST]$ .

(1)

(Total 24 marks)

45. The diagram shows the parabola  $y = (7 - x)(1 + x)$ . The points  $A$  and  $C$  are the  $x$ -intercepts and the point  $B$  is the maximum point.



Find the coordinates of  $A$ ,  $B$  and  $C$ .

(Total 4 marks)

46. The function  $f$  is given by  $f(x) = \sqrt{\ln(x - 2)}$ . Find the domain of the function.

(Total 4 marks)

47. Let  $f(x) = \sqrt{x}$ , and  $g(x) = 2^x$ . Solve the equation

$$(f^{-1} \circ g)(x) = 0.25.$$

(Total 4 marks)

48. A population of bacteria is growing at the rate of 2.3 % per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

(Total 4 marks)

49. Three of the following diagrams I, II, III, IV represent the graphs of

(a)  $y = 3 + \cos 2x$

(b)  $y = 3 \cos(x + 2)$



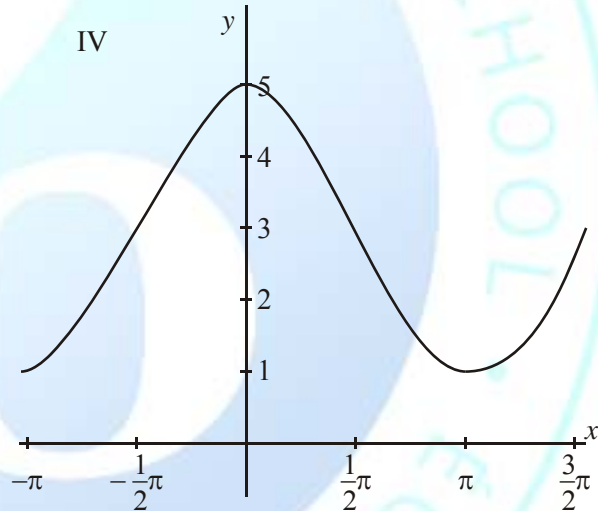
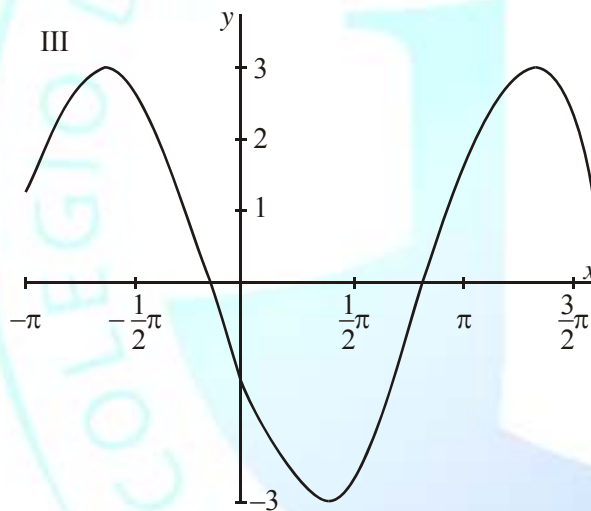
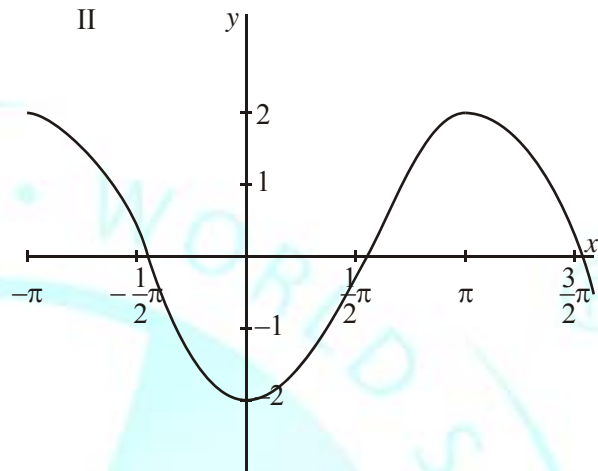
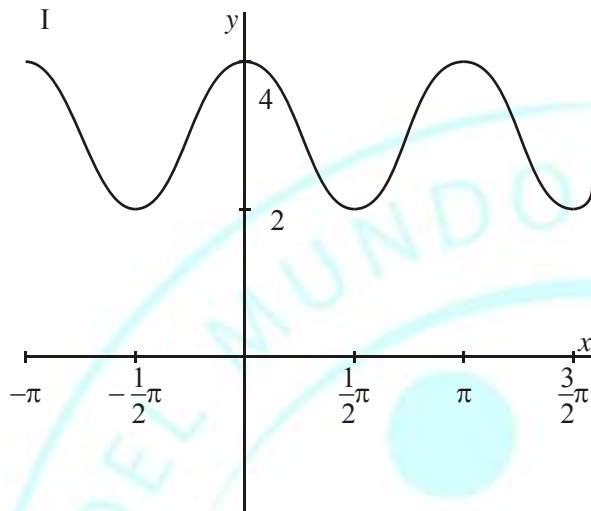
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FECHA:

(c)  $y = 2 \cos x + 3$ .

Identify which diagram represents which graph.



(Total 4 marks)

50. \$1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed \$3000.

(Total 6 marks)