1. La normal a la curva $y = \frac{k}{x} + \ln x^2$, para $x \neq 0$, $k \in \mathbb{R}$, en el punto en que $x = 2$, tiene la ecuación $3x + 2y = b$,donde $b \in \mathbb{R}$. halle **exactamente** el valor de k.

Working:

Answer:

....

(Total 6 puntos)

2. **2.** Sea *f la function,tal que,* $\int_0^3 f(x)\,dx = 8$.

(a) Deducir el valor de,

(i) $\int_0^3 2f(x)\,dx$;

(ii) $\int_0^3 (f(x)+2)\,dx$.

(b) $\int_c^d f(x-2)\,dx = 8$, escriba el valor de c y de d.

Working:

Answers:

(a) (i) ....

(ii) ....

(b) c = ..., d = ...

(Total 6 marks)

3. En un experimento, el número de bacterias "*n" en un líquido*, está dado por la fórmula $n = 650\, e^{kt}$, donde *t, el tiempo,* está en minutos y *k* es una constante. El número de bacterias se duplica cada veinte minutos. Halle,

(a) **exactamente** *k;*

(b) la razón a la que el número de bacterias se incrementa cuando $t = 90$.

Working:

Answers:

(a) ……………………………………………..

(b) ....

(Total 6 marks)

4. En la tabla se muestra algunos valores de dos funciones, *f* y *g*, así como de sus derivadas, *f′* y *g′*

x	1	2	3	4
$f(x)$	5	4	–1	3
$g(x)$	1	–2	2	–5
$f'(x)$	5	6	0	7
$g'(x)$	–6	–4	–3	4

Calcular:

(a) $\frac{\mathrm{d}}{\mathrm{d}x}(f(x) + g(x))$, cuando $x = 4$;

(b) $\int_1^3 (g'(x) + 6)\mathrm{d}x$.

Working:

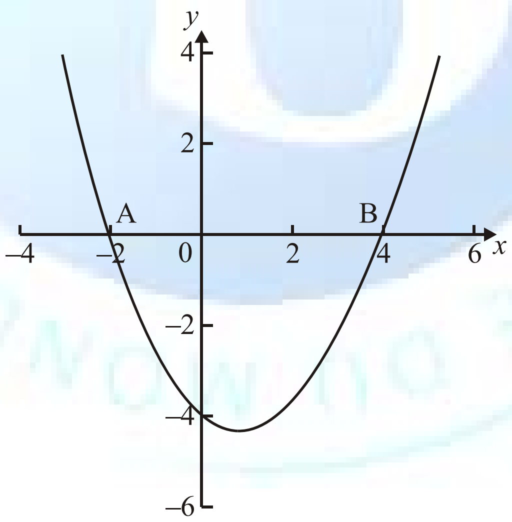
Answers:

(a) …………………………………………..

(b) ....

(Total 6 marks)

5. La ecuación de una curva puede escribise en la forma, $y = a(x - p)(x - q)$. intersecta al eje x *en* A(–2, 0) y B(4, 0). La curva $y = f(x)$ se muestra abajo.



(a) (i) Escriba el valor de p y de q.

(ii) El punto (6, 8) está en la curva, halle el valor de a.

(iii) Escriba la ecuación de la curva en la forma $y = ax^2 + bx + c$.

(5)

(b) (i) Halle $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Se dibuja una tangente en el punto P. El gradiente de esta tangente es 7. halle las coordenadas de P.

(4)

(c) La línea *L* pasa a través de B(4, 0), y es perpendicular a la tangente de la curva en el punto B.

(i) Halle la ecuación de *L*.

(ii) Halle la coordenada x (abscisa) del punto en el que *L* intersecta a la curva otra vez..

(6)

(Total 15 marks)

6. Sea $f(x) = (3x + 4)^5$.Hallar

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) ……………………………………………..

(b) ....

(Total 6 marks)

7. Use the substitution $u = x + 2$ to find $\int \frac{x^3}{(x+2)^2}\,\mathrm{d}x$.

....

....

....

....

....

....

....

....

....

....

....

....

(Total 6 marks)

8. Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{y}{\mathrm{e}^{2\theta}+1}$.

(a) Use the substitution $x = \mathrm{e}^{\theta}$ to show that

$$\int \frac{\mathrm{d}y}{y} = \int \frac{\mathrm{d}x}{x(x^2+1)}.$$

(3)

(b) Find $\int \frac{\mathrm{d}x}{x(x^2+1)}$.

(4)

(c) Hence find y in terms of θ, if $y = \sqrt{2}$ when $\theta = 0$.

(4)
(Total 11 marks)

9. Find the total area of the two regions enclosed by the curve $y = x^3 - 3x^2 - 9x + 27$ and the line $y = x + 3$.

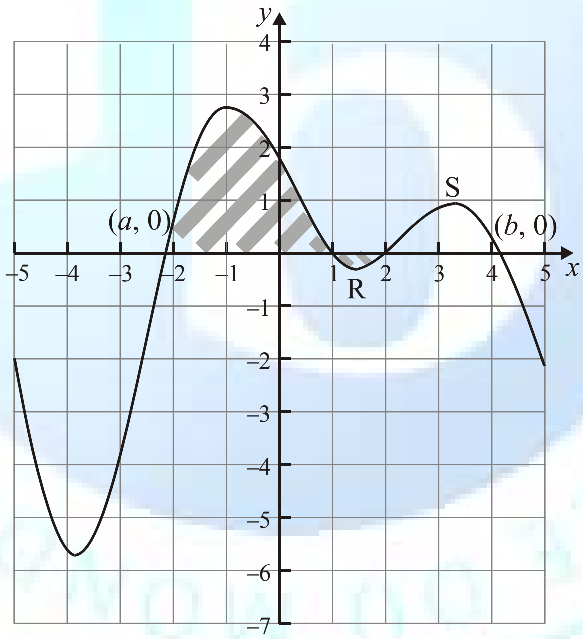
Working:

Answer:

....

(Total 6 marks)

10. Let $h(x) = (x - 2)\sin(x - 1)$ for $-5 \le x \le 5$. The curve of $h(x)$ is shown below. There is a minimum point at R and a maximum point at S. The curve intersects the x-axis at the points $(a, 0)$ $(1, 0)$ $(2, 0)$ and $(b, 0)$.



(a) Find the exact value of

(i) a;

(ii) b.

(2)

The regions between the curve and the x-axis are shaded for $a \le x \le 2$ as shown.

(b) (i) Write down an expression which represents the **total** area of the shaded regions.

(ii) Calculate this total area.

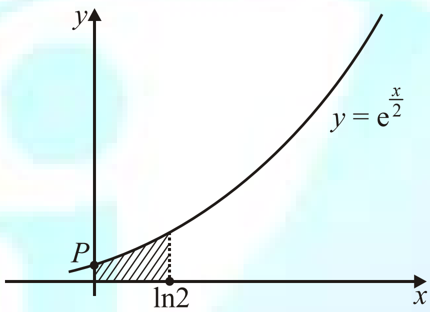
(5)

(c) (i) The y-coordinate of R is –0.240. Find the y-coordinate of S.

(ii) Hence or otherwise, find the range of values of k for which the equation $(x - 2)\sin(x - 1) = k$ has **four** distinct solutions.

(4)
(Total 11 marks)

11. The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point P, where the graph meets the y-axis.

(2)

The shaded region between the graph and the x-axis, bounded by $x = 0$ and $x = \ln 2$, is rotated through 360° about the x-axis.

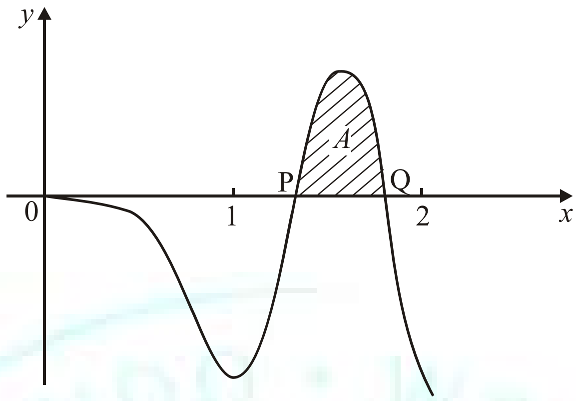
(b) Write down an integral which represents the volume of the solid obtained.

(4)

(c) Show that this volume is π.

(5)
(Total 11 marks)

12. The diagram below shows part of the graph of $f(x) = x^2 \sin(x^2 + \pi)$ and the shaded region A.



This graph crosses the x-axis at P and Q. The point P has coordinates ($\sqrt[3]{\pi}$, 0).

(a) Find the x-coordinate of Q.

(2)

(b) Use the substitution $u = x^3 + \pi$ to find $\int f(x)\,dx$.

(4)

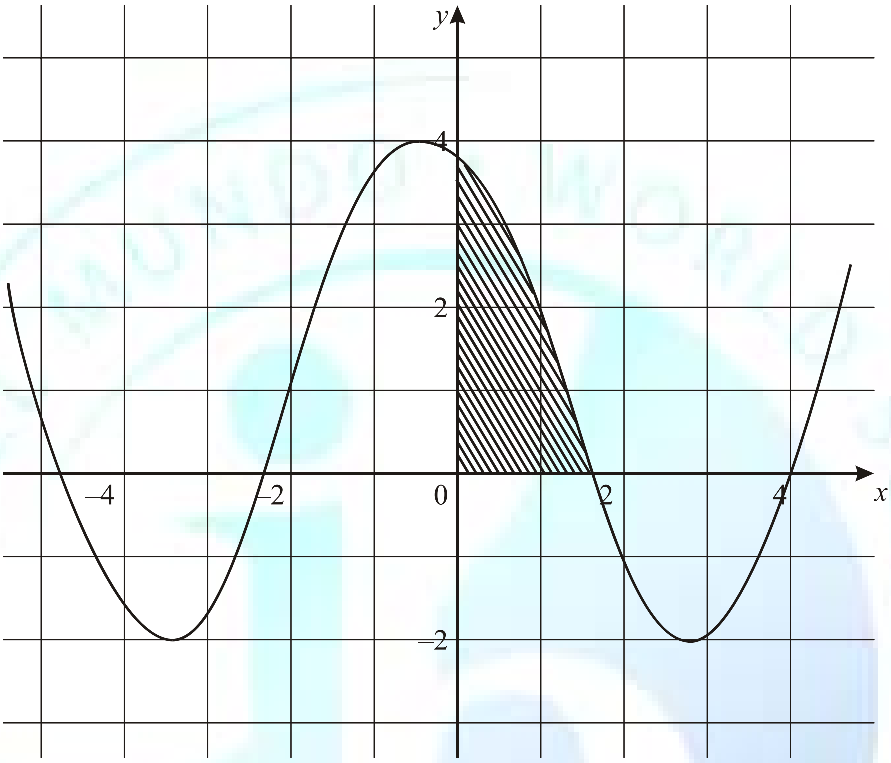
(c) Hence, using your answer to (b), find the area of the region A.

(3)
(Total 9 marks)

13. (a) Find $\int(1 + 3\sin(x + 2))\mathrm{d}x$.

(b) The diagram shows part of the graph of the function $f(x) = 1 + 3\sin(x + 2)$.

The area of the shaded region is given by $\int_0^a f(x)\mathrm{d}x$.



Find the value of a.

Working:

Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(Total 6 marks)

14. Calculate the area enclosed by the curves $y = \ln x$ and $y = e^x - \text{e}$, $x > 0$.

Working:

Answer:

…………………………………………..

(Total 6 marks)

15. Using the substitution $y = 2 - x$, or otherwise, find $\int \left(\frac{x}{2-x}\right)^2 \text{d}x$.

Working:

Answer:

…...…………………………………………….

(Total 6 marks)

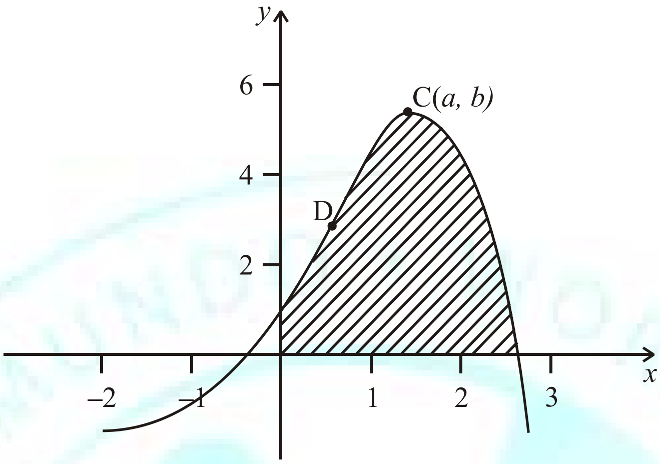
16. Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f(-\frac{\pi}{4}) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

(3)

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \le x \le 3$. The graph has a maximum turning point at C(a, b) and a point of inflexion at D.



(b) Find $\frac{dy}{dx}$.

(3)

(c) Find the **exact** value of a and of b.

(4)

(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(5)

(e) Find the area of the shaded region.

(2)
(Total 17 marks)

17. Given that $\int_1^3 g(x)\mathrm{d}x = 10$, deduce the value of

(a) $\int_1^3 \frac{1}{2} g(x)\mathrm{d}x$;

(b) $\int_1^3 (g(x)+4)\mathrm{d}x$.

Working:

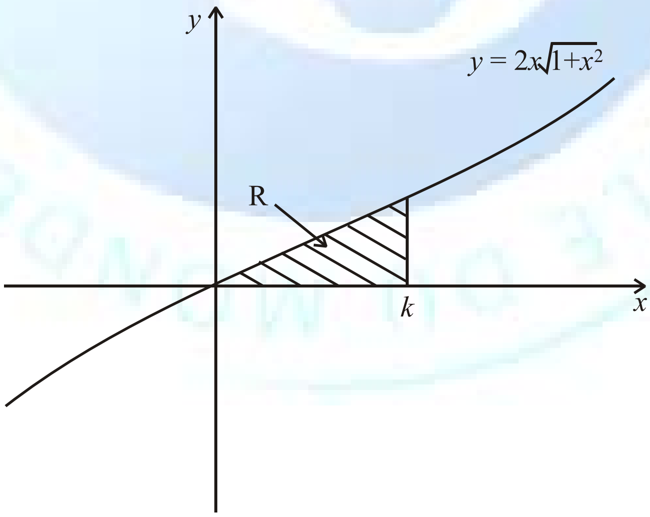
Answers:

(a) ....

(b) ....

(Total 6 marks)

18. The diagram below shows the shaded region R enclosed by the graph of $y = 2x\sqrt{1+x^2}$, the x-axis, and the vertical line $x = k$.



(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Using the substitution $u = 1 + x^2$ or otherwise, show that

$$\int 2x\sqrt{1+x^2}\,\mathrm{d}x = \frac{2}{3}(1+x^2)^{\frac{3}{2}} + c.$$

(3)

(c) Given that the area of R equals 1, find the value of k.

(3)
(Total 9 marks)

19. In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v ms^{-1} of a moving body may be written as $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, where a is a constant. Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

(5)

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station.

(5)

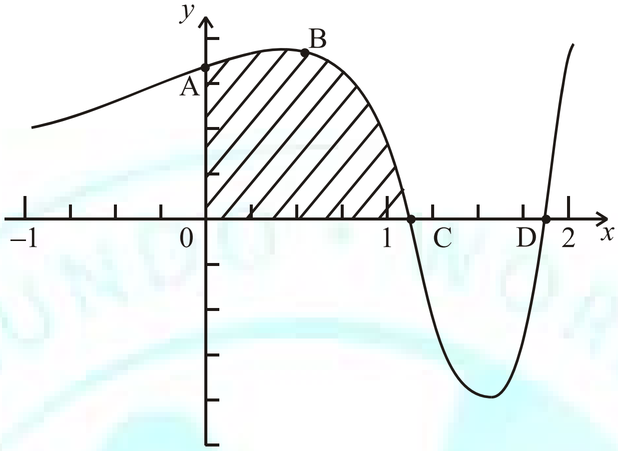
(c) Train 2 slows down so that it stops at the station. Its velocity is given by $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

(5)
(Total 15 marks)

20. The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 \leq x \leq 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A.

(2)

(b) The coordinates of C may be written as (ln k, 0). Find the **exact** value of k.

(2)

(c) (i) Write down the y-coordinate of B.

(ii) Find $\frac{dy}{dx}$.

(iii) Hence, show that at B, $x = \ln \frac{\pi}{2}$.

(6)

(d) (i) Write down the integral which represents the shaded area.

(ii) Evaluate this integral.

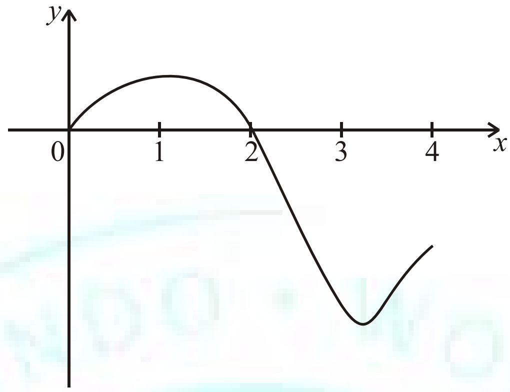
(5)

(e) (i) Copy the above diagram into your answer booklet. (There is no need to copy the shading.) On your diagram, sketch the graph of $y = x^3$.

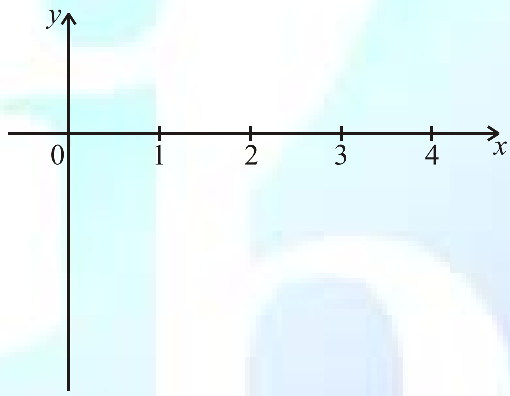
(ii) The two graphs intersect at the point P. Find the x-coordinate of P.

(3)
(Total 18 marks)

21. The diagram below shows the graph of $y_1 = f(x)$, $0 \le x \le 4$.

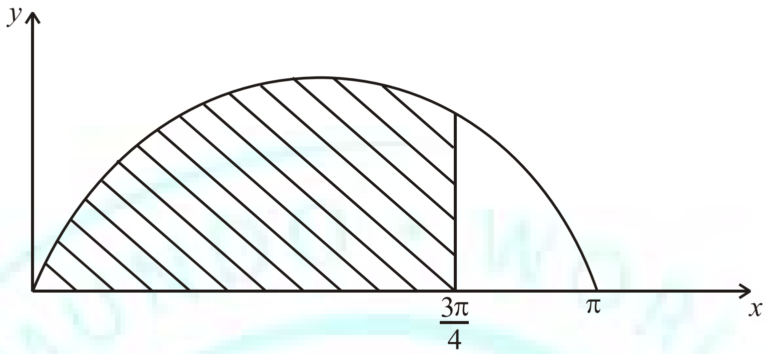


On the axes below, sketch the graph of $y_2 = \int_0^x f(t)\,dt$, marking clearly the points of inflexion.



(Total 6 marks)

22. The diagram shows part of the curve $y = \sin x$. The shaded region is bounded by the curve and the lines $y = 0$ and $x = \frac{3\pi}{4}$.



Given that $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, calculate the **exact** area of the shaded region.

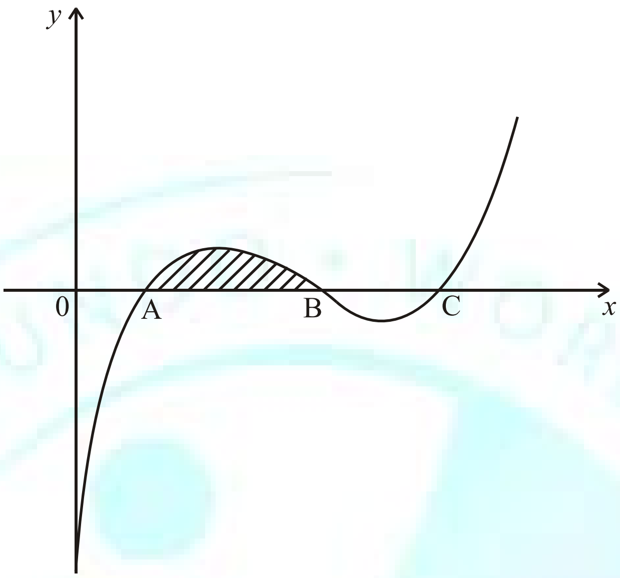
Working:

Answer:

……………………………………………..

(Total 6 marks)

23. The figure below shows part of the curve $y = x^3 - 7x^2 + 14x - 7$. The curve crosses the x-axis at the points A, B and C.



(a) Find the x-coordinate of A.

(b) Find the x-coordinate of B.

(c) Find the area of the shaded region.

Working:

Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(c) ……………………………………………..

(Total 6 marks)

24. Consider the function $f(x) = \frac{1}{x^x}$, where $x \in \mathbb{R}^+$.

(a) Show that the derivative $f'(x) = f(x)\left(\frac{1 - \ln x}{x^2}\right)$.

(3)

(b) Sketch the function $f(x)$, showing clearly the local maximum of the function and its horizontal asymptote. You may use the fact that

$$\lim_{x \to \infty} \frac{\ln x}{x} = 0.$$

(5)

(c) Find the Taylor expansion of $f(x)$ about $x = \mathrm{e}$, up to the second degree term, and show that this polynomial has the same maximum value as $f(x)$ itself.

(5)
(Total 13 marks)

25. Let $y = \sin(kx) - kx\cos(kx)$, where k is a constant.

Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = k^2 x \sin(kx)$.

(Total 3 marks)

26. Find the area enclosed by the curves $y = \frac{2}{1 + x^2}$ and $y = e^3$, given that $-3 \le x \le 3$.

Working:

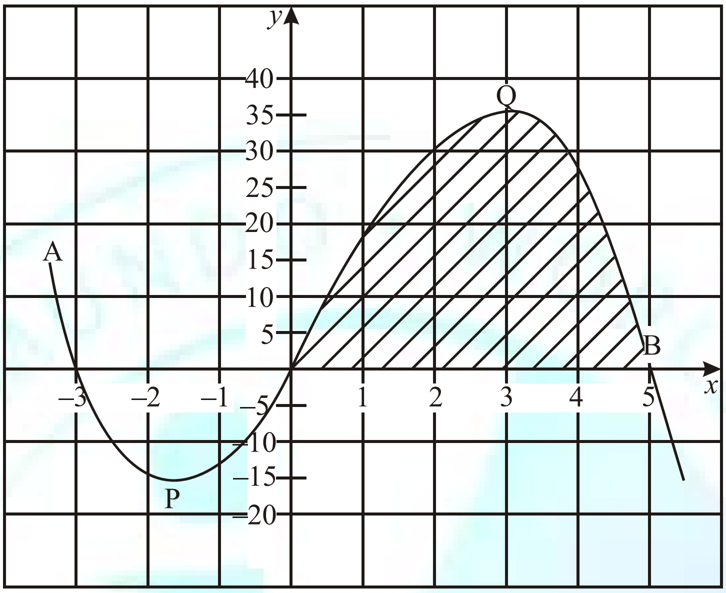
Answers:

………………………………………………..

(Total 3 marks)

27. The diagram below shows part of the graph of the function

$$f : x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the x-axis at A(–3,0), B(5,0) and the origin, O. There is a minimum point at P and a maximum point at Q.

(a) The function may also be written in the form $f : x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a;

(ii) b.

(2)

(b) Find

(i) $f'(x)$;

(ii) the **exact** values of x at which $f'(x) = 0$;

(iii) the value of the function at Q.

(7)

(c) (i) Find the equation of the tangent to the graph of f at O.

(ii) This tangent cuts the graph of f at another point. Give the x-coordinate of this point.

(4)

(d) Determine the area of the shaded region.

(2)
(Total 15 marks)

28. Let $f(t) = t^{\frac{1}{3}}\left(1 - \frac{1}{2t^{\frac{5}{3}}}\right)$. Find $\int f(t)\,\mathrm{d}t$.

Working:

Answers:

……………………………………………..

(Total 3 marks)

29. Find

(a) $\int \sin(3x+7)\,\mathrm{d}x$;

(b) $\int e^{-4x}\,\mathrm{d}x$.

Working:

Answers:

……………………………………………..

(Total 4 marks)

30. The function f is given by $f(x) = \dfrac{\ln 2x}{x}$, $x > 0$.

(a) (i) Show that $f'(x) = \dfrac{1 - \ln 2x}{x^2}$.

Hence

(ii) prove that the graph of f can have only one local maximum or minimum point;

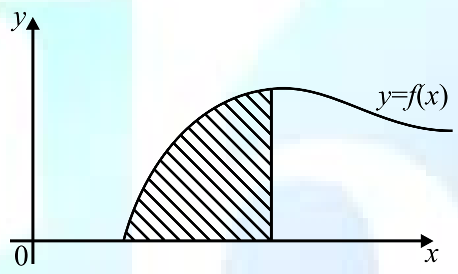
(iii) find the coordinates of the maximum point on the graph of f.

(6)

(b) By showing that the second derivative $f''(x) = \dfrac{2\ln 2x - 3}{x^3}$ or otherwise, find the coordinates of the point of inflexion on the graph of f.

(6)

(c) The region S is enclosed by the graph of f, the x-axis, and the vertical line through the maximum point of f, as shown in the diagram below.



(i) Would the trapezium rule overestimate or underestimate the area of S? Justify your answer by drawing a diagram or otherwise.

(3)

(ii) Find $\int f(x)\,dx$, by using the substitution $u = \ln 2x$, or otherwise.

(4)

(iii) Using $\int f(x)\,dx$, find the area of S.

(4)

(d) The Newton-Raphson method is to be used to solve the equation $f(x) = 0$.

(i) Show that it is not possible to find a solution using a starting value of $x_1 = 1$.

(3)

(ii) Starting with $x_1 = 0.4$, calculate successive approximations $x_2, x_3, \ldots$ for the root of the equation until the absolute error is less than 0.01. Give all answers correct to **five** decimal places.

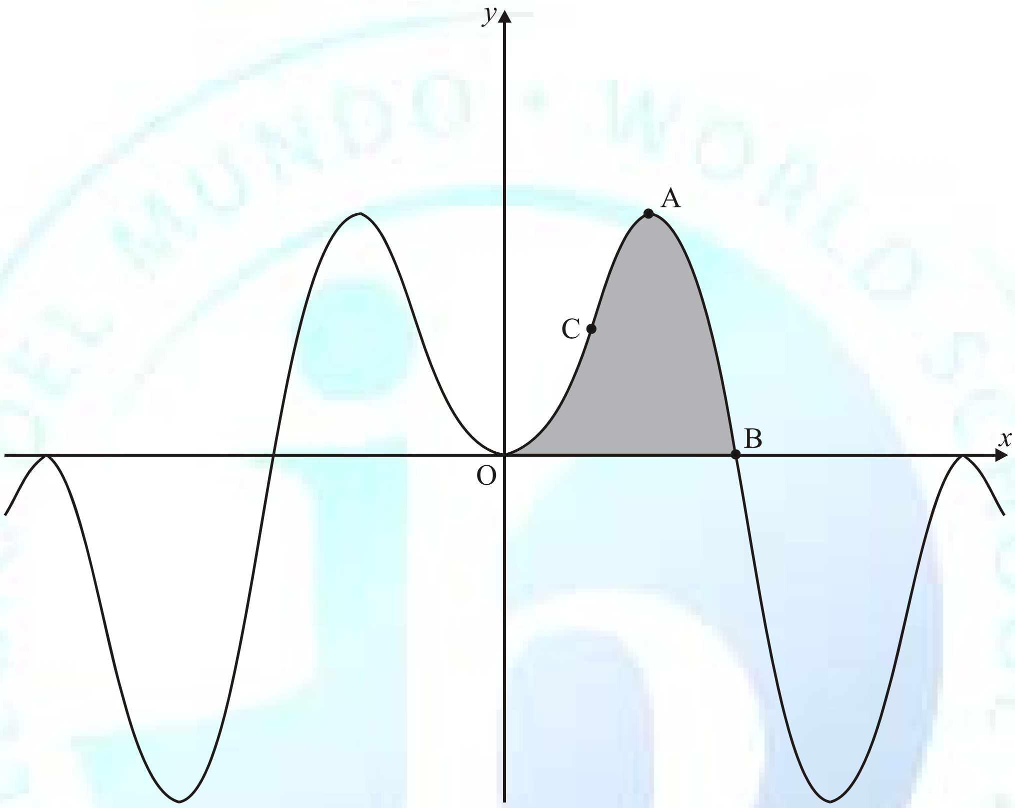
(4)
(Total 30 marks)

31. In this part of the question, radians are used throughout.

The function f is given by

$$f(x) = (\sin x)^2 \cos x.$$

The following diagram shows part of the graph of $y = f(x)$.



The point A is a maximum point, the point B lies on the x-axis, and the point C is a point of inflexion.

(a) Give the period of f. **(1)**

(b) From consideration of the graph of $y = f(x)$, find **to an accuracy of one significant figure** the range of f. **(1)**

(c) (i) Find $f'(x)$.

(ii) Hence show that at the point A, $\cos x = \sqrt{\frac{1}{3}}$.

(iii) Find the exact maximum value. **(9)**

(d) Find the exact value of the x-coordinate at the point B. **(1)**

(e) (i) Find $\int f(x)\,dx$.

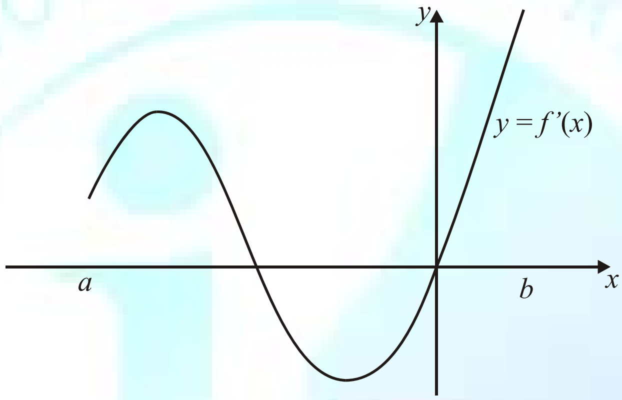
(ii) Find the area of the shaded region in the diagram.

(4)

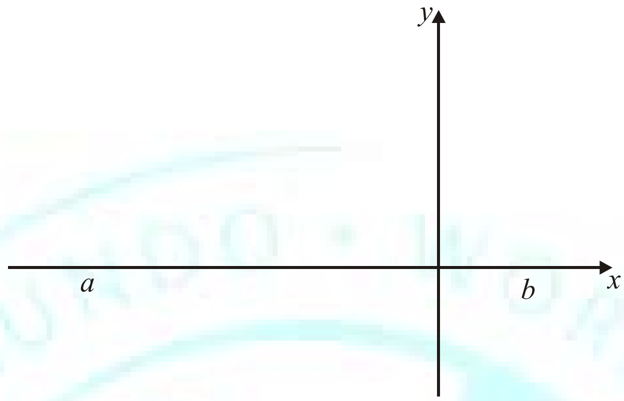
(f) Given that $f''(x) = 9(\cos x)^3 - 7\cos x$, find the x-coordinate at the point C.

(4)
(Total 20 marks)

32. The diagram shows a sketch of the graph of $y = f'(x)$ for $a \le x \le b$.



On the grid below, which has the same scale on the x-axis, draw a sketch of the graph of $y = f(x)$ for $a \le x \le b$, given that $f(0) = 0$ and $f(x) \ge 0$ for all x. On your graph you should clearly indicate any minimum or maximum points, or points of inflexion.



Working:

(Total 3 marks)

33. (a) Sketch and label the graphs of $f(x) = e^{-x^2}$ and $g(x) = e^{x^2} - 1$ for $0 \le x \le 1$, and shade the region A which is bounded by the graphs and the y-axis.

(3)

(b) Let the x-coordinate of the point of intersection of the curves $y = f(x)$ and $y = g(x)$ be p.

Without finding the value of p, show that

$$\frac{p}{2} < \text{area of region } A < p.$$

(4)

(c) Find the value of p correct to four decimal places.

(2)

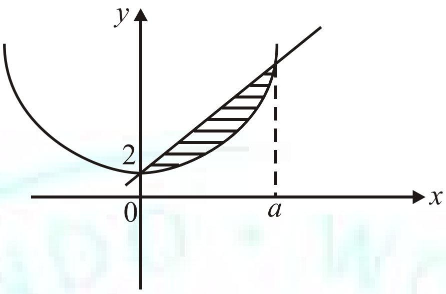
(d) Express the area of region A as a definite integral and calculate its value.

(3)

(Total 12 marks)

34. The area of the enclosed region shown in the diagram is defined by

$$y \geq x^2 + 2,\ y \leq ax + 2, \text{ where } a > 0.$$



This region is rotated 360° about the x-axis to form a solid of revolution. Find, in terms of a, the volume of this solid of revolution.

Working:

Answers:

…………………………………………….

(Total 4 marks)

35. A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height h metres of the rock-climber after t seconds of the fall is given by:

$H = 50 - 5t^2$, $\quad 0 \leq t \leq 2$

$H = 90 - 40t + 5t^2$, $\quad 2 \leq t \leq 5$

(a) Find the height of the rock-climber when $t = 2$.

(1)

(b) Sketch a graph of h against t for $0 \leq t \leq 5$.

(4)

(c) Find $\frac{dh}{dt}$ for:

(i) $0 \leq t \leq 2$

(ii) $2 \le t \le 5$

(2)

(d) Find the velocity of the rock-climber when $t = 2$.

(2)

(e) Find the times when the velocity of the rock-climber is zero.

(3)

(f) Find the minimum height of the rock-climber for $0 \le t \le 5$.

(3)
(Total 15 marks)

36. Given that $f(x) = (2x + 5)^3$ find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) …………………………………………….

(b) ....

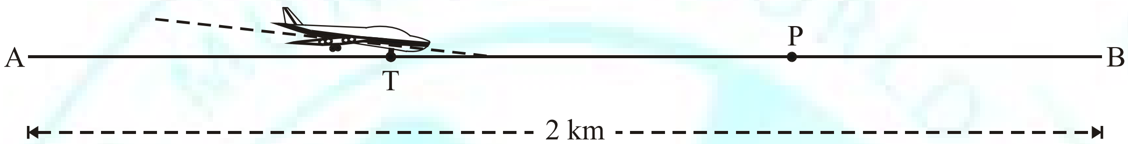
(Total 4 marks)

37. A curve with equation $y = f(x)$ passes through the point (1, 1). Its gradient function is $f'(x) = 2x + 3$.

Find the equation of the curve.

(Total 4 marks)

38. The main runway at *Concordville* airport is 2 km long. An aeroplane, landing at *Concordville,* touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the aeroplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The aeroplane touches down 800 m from A, (*i.e.* $c = 800$).

(i) Find the distance travelled by the aeroplane in the first 5 seconds after touchdown. **(2)**

(ii) Write down an expression for the velocity of the aeroplane at time t seconds after touchdown, and hence find the velocity after 5 seconds. **(3)**

The aeroplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker;

(2)

(iv) the distance from P to A.

(3)

(b) Show that if the aeroplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway.

(5)
(Total 15 marks)

39. Let $f(x) = \ln|x^5 - 3x^2|$, $-0{\cdot}5 < x < 2$, $x \neq a$, $x \neq b$; (a, b are values of x for which $f(x)$ is not defined).

(a) (i) Sketch the graph of $f(x)$, indicating on your sketch the number of zeros of $f(x)$. Show also the position of any asymptotes.

(2)

(ii) Find all the zeros of $f(x)$, (that is, solve $f(x) = 0$).

(3)

(b) Find the **exact** values of a and b.

(3)

(c) Find $f(x)$, and indicate clearly where $f(x)$ is not defined.

(3)

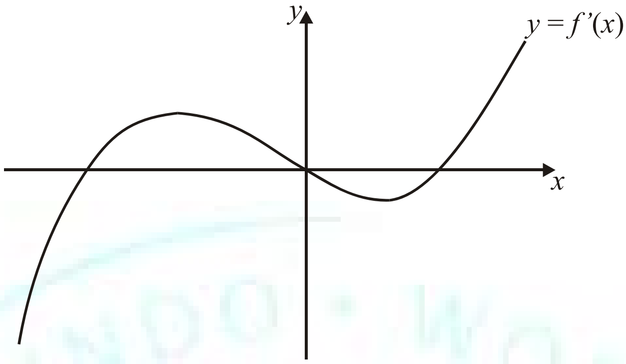
(d) Find the **exact** value of the x-coordinate of the local maximum of $f(x)$, for $0 < x < 1.5$. (You may assume that there is no point of inflexion.)

(3)

(e) **Write down** the definite integral that represents the area of the region **enclosed** by $f(x)$ and the x-axis. (Do **not** evaluate the integral.)

(2)
(Total 16 marks)

40. The diagram shows the graph of $y = f'(x)$.



Indicate, and label clearly, **on the graph**

(a) the points where $y = f(x)$ has minimum points;

(b) the points where $y = f(x)$ has maximum points;

(c) the points where $y = f(x)$ has points of inflexion.

Working:

(Total 3 marks)

41. Let $f(x) = x^3$.

(a) Evaluate $\dfrac{f(5+h) - f(5)}{h}$ for $h = 0.1$.

(b) What number does $\dfrac{f(5+h) - f(5)}{h}$ approach as h approaches zero?

Working:

Answers:

(a) …………………………………………..

(b) ………………………………………………………

(Total 4 marks)

42. Using the substitution $u = \dfrac{1}{2}x + 1$, or otherwise, find the integral

$$\int x\sqrt{\frac{1}{2}x+1}\ \mathrm{d}x.$$

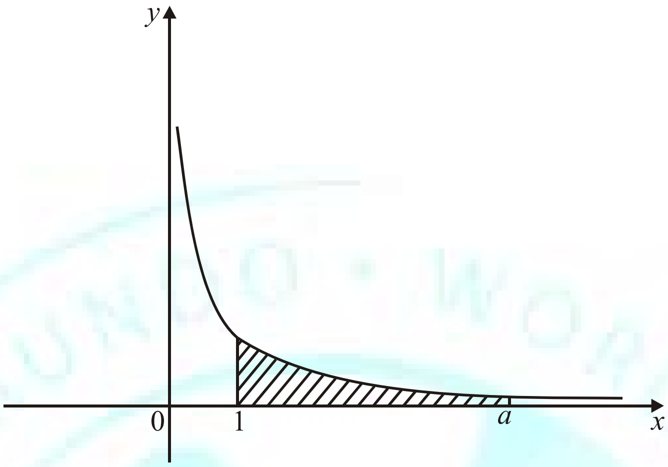
Working:

Answers:

…………………………………………..

(Total 4 marks)

43. The diagram shows part of the graph of $y = \frac{1}{x}$. The area of the shaded region is 2 units.



Find the exact value of a.

Working:

Answers:

....

(Total 4 marks)

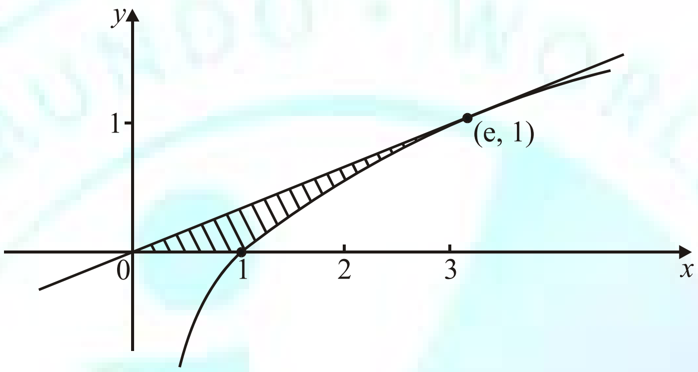
44. (a) Find the equation of the tangent line to the curve $y = \ln x$ at the point (e, 1), and verify that the origin is on this line.

(4)

(b) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$.

(2)

(c) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line in part (a), and the line $y = 0$.



Use the result of part (b) to show that the area of this region is $\frac{1}{2}e - 1$.

(4)
(Total 10 marks)

45. The area between the graph of $y = e^x$ and the x-axis from $x = 0$ to $x = k$ ($k > 0$) is rotated through 360° about the x-axis. Find, in terms of k and e, the volume of the solid generated.

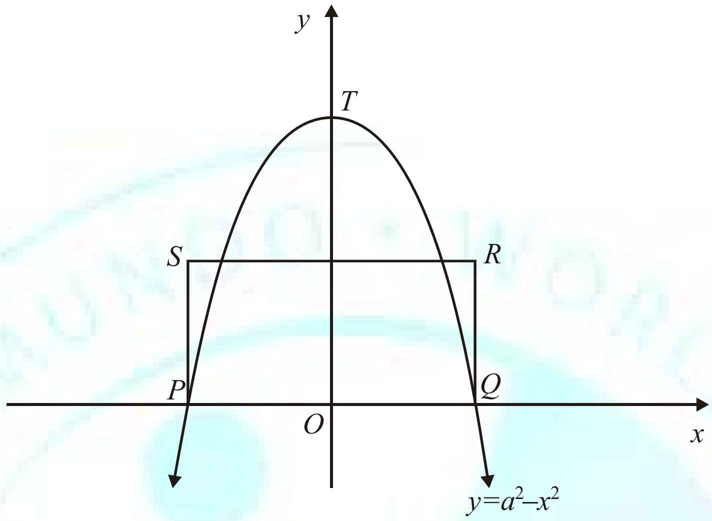
Working:

Answers:

....

(Total 4 marks)

46. In the diagram, PTQ is an arc of the parabola $y = a^2 - x^2$, where a is a positive constant, and $PQRS$ is a rectangle. The area of the rectangle $PQRS$ is equal to the area between the arc PTQ of the parabola and the x-axis.



Find, in terms of a, the dimensions of the rectangle.

Working:

Answers:

....

(Total 4 marks)